

Turbo Trellis Coded Modulation (TTCM) with Imperfect Phase Reference

Faz Bozulmalı Ortamlarda Turbo Kafes Kodlamalı Sistemler

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ABSTRACT

In this paper, we investigate the performance of Turbo Trellis Coded Modulation (TTCM) over AWGN, and Rician channels and assume that phase disturbance is available. As we know Turbo Trellis Coded Modulation (TTCM) is similar to binary turbo codes, but employs Trellis Coded Modulation (TCM) codes which include multi-dimensional codes. The combination of turbo codes with trellis codes leads to a strighthforward encoder structure, and allows iterative decoding as binary turbo decoder. However, iterative Turbo Decoder needs to be adapted to the decoding of TCM codes. Here, we investigate TTCM for 8PSK for several Rician parameter K and effective signal-to-noise ratio in the carrier tracking loop α . Thus, our results will reflect the degredations both due to the effects of the fading on the amplitude of the received signal and of a noisy carrier reference.

Keywords- Turbo Trellis Coded Modulation, Rician Channel, Imperfect Phase Reference

ÖZET

Bu makalede Turbo Kafes Kodlamalı (TKK) sistemlerin AWGN ve Rician ortamlarda faz bozulması durumunda hata balarıymı elde edilmiştir. TKK yapısı ikili Turbo modülasyonu gibidir ancak TKK çok boyutludur. Turbo kodlamanın Kafes Kodlama ile birlikte kulanımı kodlayıcı yapısının deđimini gerektirir. Alıcıda ise ikili kodlayıcılarda olduđu gibi iteratif kod çözücü yapısı vardır. Burada 8PSK modülasyonda TKK farklı Rician katsayısı K , faz bozulma katsayısı α için benzetim sonuçları elde edilmiştir. Elde edilen sonuçlar hem genlikte hem de faz daki bozulmayı içermektedir.

Anahtar Kelimeler: Turbo Kafes Kodlama, Rician Kanal, Faz Bozulması

1. Introduction

Turbo codes are a new class of error correction codes that were introduced a long with a practical decoding algorithm in [1]. The importance of turbo codes is that they enable reliable communications with power efficiencies close to the theoretical limit predicted by Claude Shannon. Since their introduction, turbo codes have been proposed for low-power applications such as deep-space and satellite communications, as well as for interference limited applications such as third generation cellular and personal communication services.

Since Turbo codes use convolutional codes as their constituent codes, a natural extension of the Turbo concept, which improves bandwidth efficiency is its application to systems using TCM. As in [2], the main principle of turbo codes is applied to TCM by retaining the important properties and advantages of both of their structures. Essentially, TCM codes can be seen as systematic feedback convolutional codes followed by one (or more for multi dimensional codes) signal mapper(s). Just as binary turbo codes use a parallel concatenation of two binary recursive convolutional encoders, we have concatenated two recursive TCM encoders as in [2][3], and adapted the interleaving and puncturing.

This paper, presents the performance of TTCM for different Rician parameter K and different phase reference in the carrier tracking loop α and overall scheme of the system is shown in Figure 1. First of all we will explain Turbo Trellis Encoder structure. After giving a brief information for Rician channels with imperfect phase reference, we will explain the iterative symbol by symbol log-MAP algorithm for Turbo Trellis structure. Then performance of Turbo Trellis coded signals are evaluated for Rician fading parameter K , and effective signal-to-noise ratio in the carrier tracking loop α , iteration number, data block size N and signal-to-noise ratio SNR.

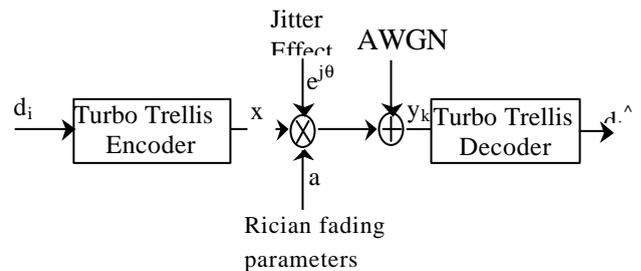


Figure 1. General Block Diagram of TTCM Model

2. Turbo Trellis Encoder

The most important characteristic of turbo codes is their simple use of recursive systematic component codes in a parallel concatenation scheme. Pseudo-random bit-wise interleaving between encoders ensures a small bit-error probability. In [2], Ungerboeck codes (and multidimensional TCM codes) have been employed as building blocks in a turbo coding scheme in a similar way as binary codes were used [1].

We will use the scheme of encoder in Figure 2, it was presented in [2], to clarify the operations which we use in our simulations. Here $\tilde{m} = m = 2$ (\tilde{m} = coded input bits, m = total input bits), $N = 6$ (N = frame size), and 8-PSK signalling. The Mapper structure is shown in Figure 3. The six long sequence $(d_1, d_2, \dots, d_6) = (00, 01, 11, 10, 00, 11)$ of information bit pairs is encoded in an Ungerboeck style encoder to yield the 8-PSK sequence $(0, 2, 7, 5, 1, 6)$. The information bits are interleaved on a pairwise basis and encoded again into the sequence $(6, 7, 0, 3, 0, 4)$. We deinterleave the second encoder's output symbols to ensure that the ordering of the two information bits partly defining each symbol corresponds to that of the first encoder, i.e., we now have the sequence $(0, 3, 6, 4, 0, 7)$. Finally, we transmit the first symbol of the first encoder, the second symbol of the second encoder, the third symbol of the first encoder, the fourth symbol of the second encoder, etc, $(0, 3, 7, 4, 1, 7)$. By using interleaver

and de-interleaver for the second encoder, each symbol index k before the selector has the property of being associated with input information bit group index k .

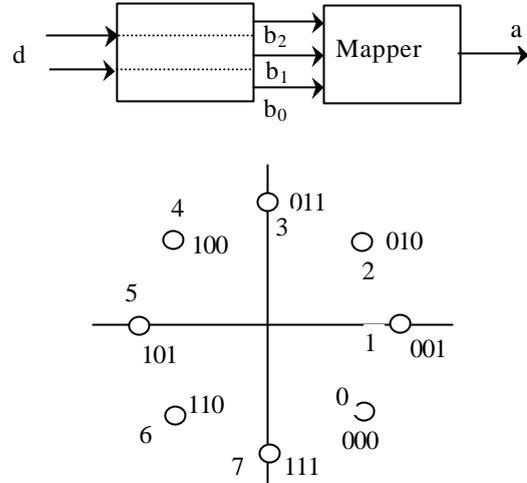


Figure 3. Set Partitioning for 8-PSK

The interleaver must map even positions to even positions and odd positions to odd ones. For 8-PSK with $m=2$, this ensure that each information bit influences either the state of the upper or lower encoder but never both.

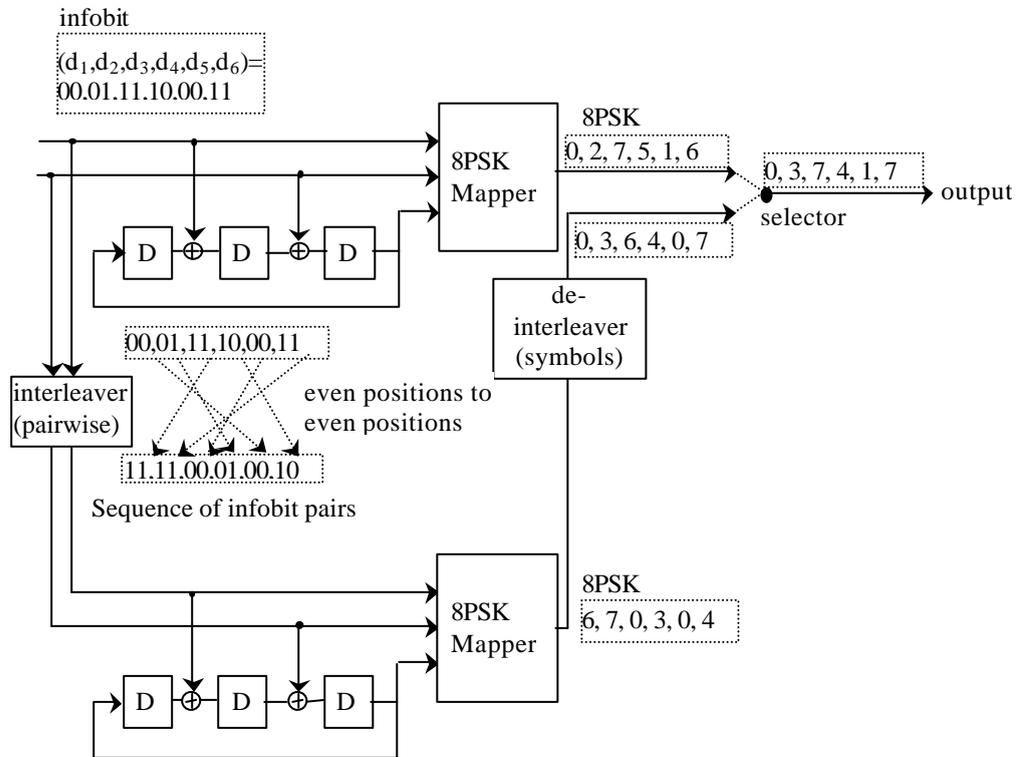


Figure 2. TTCM Encoder for 8-PSK

4. Channel Model

Here, the main emphasis is on the performance of the Turbo Trellis Coded Modulation (TTCM) in fading environment modelled by Rician probability density functions. The combined effects of fading and the non-ideal coherent receiver on the phase of the received signal is taken into account. Thus our results reflect the degradations both due the effects of fading on the amplitude of the received signal and of a noisy carrier reference. The block diagram of the considered system operating over imperfect phase reference and various fading environments are shown in Figure 1. At the channel output y_k is,

$$y_k = \mathbf{r}_k x_k e^{j\mathbf{q}_k} + n_k \quad (1)$$

n_k is Gaussian Noise where the noise variance is

$$\sigma^2 = \frac{N_o}{2E_s}, \quad \mathbf{r}_k \text{ is fading amplitude and the term}$$

$e^{j\mathbf{q}_k}$ is a unit vector where \mathbf{q}_k represents the phase noise as mentioned in [4], which is assumed to have Tikhonov pdf given by,

$$p(\mathbf{q}_k) = \frac{e^{\mathbf{a} \cos(\mathbf{q}_k)}}{2\mathbf{p}_o(\mathbf{a})} \quad \left| \mathbf{q}_k \right| \leq \mathbf{p} \quad (2)$$

In this paper, we investigate bit error performance of TTCM over Rician environment with imperfect phase reference. Rician probability density function can be written as ,

$$P(\mathbf{r}) = 2\mathbf{r}(1+K) e^{(-\mathbf{r}^2(1+K) - K)} I_0 \left[2\mathbf{r}\sqrt{K(1+K)} \right] \quad (3)$$

where I_0 is the modified Bessel function of the first

kind, order zero and K is fading parameter.

5. Turbo Trellis Decoder

The Turbo Trellis Decoder is similar to that used to decode binary turbo codes, except there is a difference passed from one decoder to the other, and the treatment of the very first decoding step.

In binary turbo coding , the decoder's output can be split into three parts. These parts are systematic component, a priori component, and extrinsic component [5]. But only the extrinsic component may be given to the next decoder; otherwise, information will be used more than once in the next decoder [1].

Here, for the Turbo Trellis Decoder the systematic information is transmitted together with parity information in the same symbol. However, we can split the decoder output into two different components, first one is a priori and the second is extrinsic and systematic together.

5.1 Metric Calculation

Matrix calculation was used in the very first decoding stage as in [2]. We have relied on the fact that if the upper decoder sees a group of n punctured symbols, we have embedded the systematic information in the a priori input (Figure 4). Before the first decoding pass off the upper decoder, we need to set the a priori information to contain the systematic information for the * transitions, where the symbol is transmitted partly by the information group d_k , but also the unknown parity bit $b_k^{0,*} \in \{0,1\}$ produced by the other encoder. We set the a priori information, by applying the mixed Bayes' rule, to what it is assumed that $Pr\{b_k^{0,*}=j|d_k\} = Pr\{b_k^{0,*}=j\}=1/2$ and $\mathbf{y}_k=(y_k^0, \dots, y_k^{(n-1)})$

$$\begin{aligned} Pr\{d_k = i | \mathbf{y}_k\} &= const \cdot p(\mathbf{y}_k | d_k = i) \\ &= const \cdot \sum_{j \in \{0,1\}} p(\mathbf{y}_k, b_k^{0,*} = j | d_k = i) \\ &= \frac{const}{2} \cdot \sum_{j \in \{0,1\}} p(\mathbf{y}_k | d_k = i, b_k^{0,*} = j) \end{aligned} \quad (4)$$

if the receiver observes N set of n noisy symbols, where n such symbols are associated with each step in the trellis. In Equation 10, it is not necessary to calculate the value of the constant since the value of $Pr\{d_k=i|\mathbf{y}_k\}$ can be determined by dividing the summation $\sum_{j \in \{0,1\}}$ by its sum over all i (normalization). If the upper decoder is not at the a * transition, then we set $Pr\{d_k=i\}$ to $1/2^m$ where m is the number of Turbo Trellis encoder input.

5.2. Decoder

To illustrate the performance in the log domain, consider the *Jacobian Logarithm*:

$$\begin{aligned} \ln(e^x + e^y) &= \max(x,y) + \ln(1 + \exp\{-|y-x|\}) \\ &= \max(x,y) + f_c(|x-y|) \end{aligned} \quad (5)$$

First of all the state transitions must be calculated by the given formulation below,

$$\begin{aligned} \mathbf{g}_i(y_k, M', M) &= p(y_k | d_k = i, S_k = M, S_{k-1} = M') \\ &= \cdot q(d_k = i | S_k = M, S_{k-1} = M') \\ &= \cdot Pr\{S_k = M | S_{k-1} = M'\} \end{aligned} \quad (6)$$

$q(d_k = i | S_k = M, S_{k-1} = M')$ is either zero or one depending on whether encoder input $i \in \{0,1, \dots, 2^m-1\}$ is associated with the transition from state $S_{k-1}=M'$ to $S_k=M$ or not. In the last component of Equation 12, we use the a priori information

$$\begin{aligned} &Pr\{S_k = M | S_{k-1} = M'\} \\ &= \begin{cases} Pr\{d_k = 0\}, & \text{if } q(d_k = 0 | S_k = M, S_{k-1} = M') = 1 \\ Pr\{d_k = 1\}, & \text{if } q(d_k = 1 | S_k = M, S_{k-1} = M') = 1 \end{cases} \\ &= \begin{cases} Pr\{d_k = 2^m - 1\}, & \text{if } q(d_k = 2^m - 1 | S_k = M, S_{k-1} = M') = 1 \end{cases} \\ &= Pr\{d_k = j\} \end{aligned} \quad (7)$$

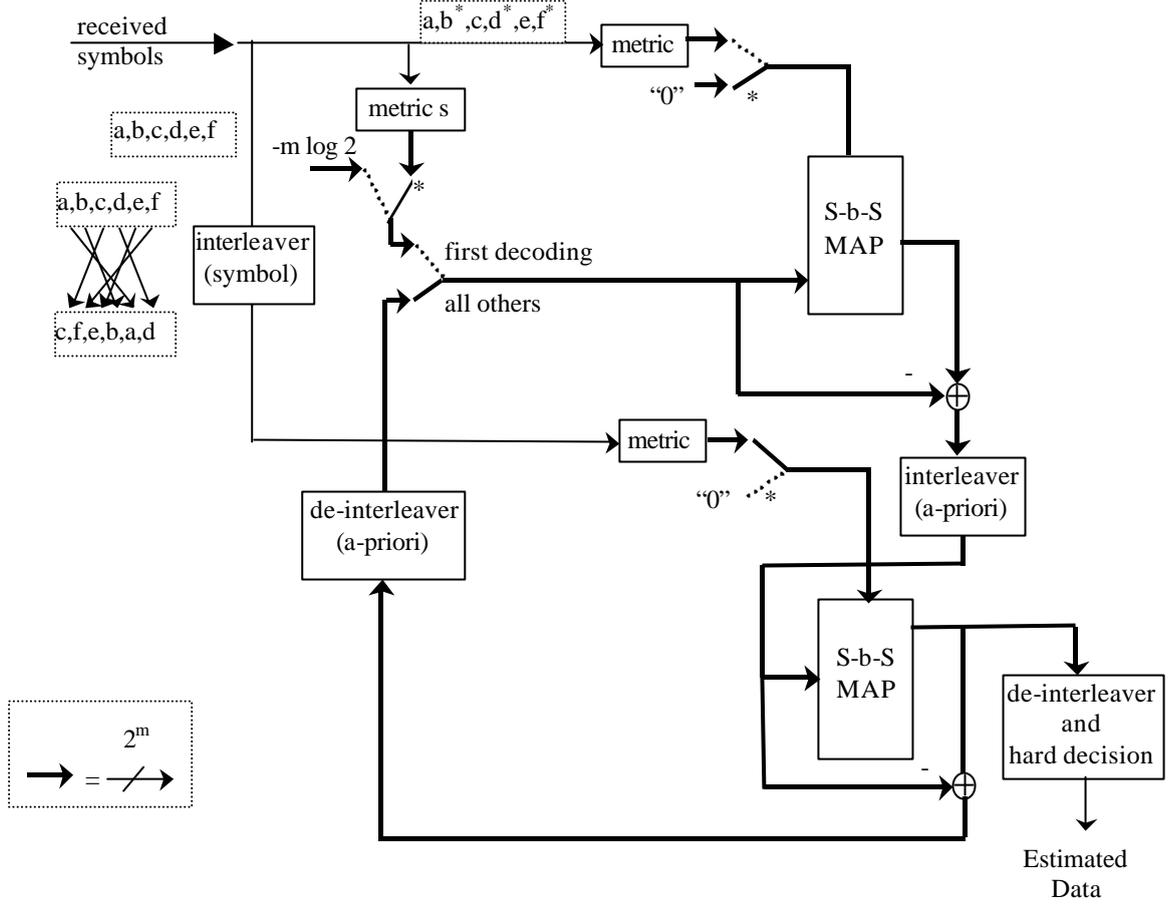


Figure 4. TTCM Decoder

where $j:q(d_k=j|S_k=M, S_{k-1}=M')=1$. If there does not exist a j such that $q(d_k=j|S_k=M, S_{k-1}=M')=1$, then $\Pr\{S_k=M|S_{k-1}=M'\}$ is set to zero

Now let $\bar{g}_i(s_k \rightarrow s_{k+1})$ be the natural logarithm of $g_i(s_k \rightarrow s_{k+1})$

$$\bar{g}_i(s_k \rightarrow s_{k+1}) = \ln g_i(s_k \rightarrow s_{k+1}) \quad (8)$$

Now let $\bar{a}(s_k)$ be the natural logarithm of $a(s_k)$,

$$\begin{aligned} \bar{a}(s_k) &= \ln a(s_k) \\ &= \ln \left\{ \sum_{s_{k-1} \in A} \exp[\bar{a}(s_{k-1}) + \bar{g}_i(s_{k-1} \rightarrow s_k)] \right\} \quad (9) \end{aligned}$$

where A is the set of states s_{k-1} that are connected to the state s_k .

Now let $\bar{b}(s_k)$ be the natural logarithm of $b(s_k)$,

$$\begin{aligned} \bar{b}(s_k) &= \ln b(s_k) \\ &= \ln \left\{ \sum_{s_{k+1} \in B} \exp[\bar{b}(s_{k+1}) + \bar{g}_i(s_k \rightarrow s_{k+1})] \right\} \quad (10) \end{aligned}$$

where B is the set of states s_{k+1} that are connected to state s_k . Therefore, the desired output of the MAP decoder is

$$P_r[d_k=i|\underline{y}] = \text{const} \sum_M \sum_{M'} [\bar{g}_i(y_k, M', M) + \bar{a}_{k-1}(M') + \bar{b}_k(M)] \quad (11)$$

$\forall i \in \{0, \dots, 2^m-1\}$. The constant can be eliminated by normalizing the sum of above formulation over all i to unity.

6. Simulation Results

In the encoder structure as shown in Figure 2, we have two information inputs, each with 1024 bit frame size. To create parity bits, both encoders are constituted of three memories. 8PSK mappers were used. At the decoder, the received signal's quadrature and inphase coordinates is detected by using the noisy phase value. The transmitted signal is estimated at the Turbo-Trellis decoder as shown in Figure 4. Here, we simulate TTCM system over AWGN, Rician with imperfect phase reference. We obtain the simulation curves for different K, α and SNR values. The first figure, Figure 5, shows performance of TTCM signals for $K=\infty, 20$ and 10 with ideal CSI. The

next figure, Figure 6, gives the performance of TTCM signals for $K=\infty$ and $\alpha=20$ and 10 dB. In Figure 7, TTCM performance is simulated for $K=20$ and $\alpha=20$ and 10 dB. And Figure 8, shows the performance of TTCM signals for $K=10$ and $\alpha=20$ and 10 dB. It is clear that for a constant iteration number, as K increases, performance improves for the same SNR values. To emphasize the importance of imperfect phase effect, the performance of the considered scheme is simulated with various α values and K . For $\alpha=10, 20, \infty$ dB and $K=10, 20, \infty$ SNR value, as iteration number increases, performance gets better. It is clear that phase jitter distortion is effective for Rician fading. When the performance results obtained in Figure 5 through Figure 8 are compared, the degradation of error performance due to phase distortion can easily be seen for all SNR and K values.

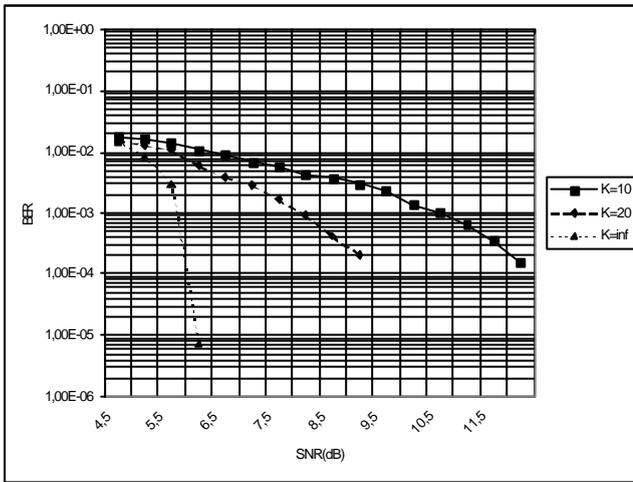


Figure 5. 8th iteration performance in TTCM for $K=10, 20, \infty$, $\alpha=\infty$ and $N=1024$.

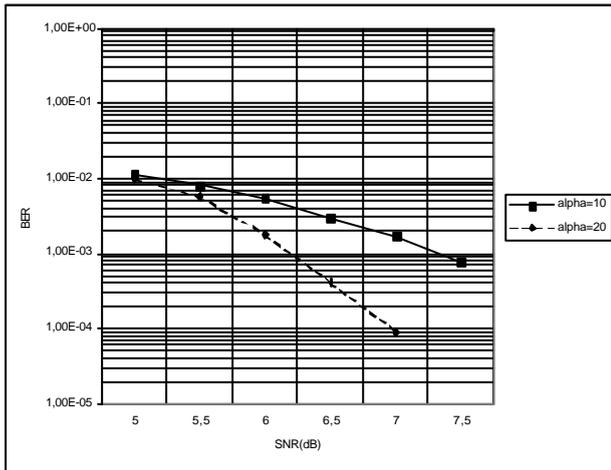


Figure 6. 8th iteration performance in TTCM for $K=\infty$, $\alpha=10, 20$ and $N=1024$.

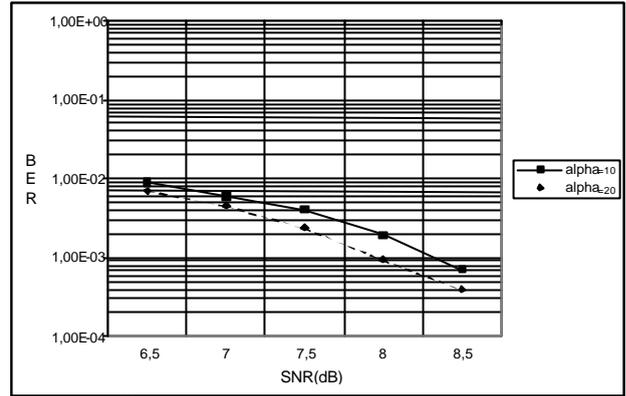


Figure 7. 8th iteration performance in TTCM for $K=20$, $\alpha=10, 20$ and $N=1024$.

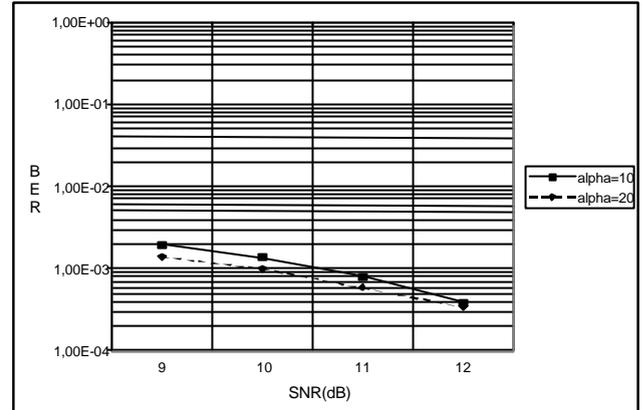


Figure 8. 8th iteration performance in TTCM for $K=10$, $\alpha=10, 20$ and $N=1024$.

7. Conclusion

In this paper we have shown the performance of turbo trellis codes over Rician fading channels with jitter effect. As an example, the jitter performance of turbo trellis coded modulated signals are simulated with different coding parameter K , effective signal-to-noise ratio in the carrier tracking loop α , iteration number and data block size N .

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