EM-Based MAP Channel Estimation and Data Detection for Downlink MC-CDMA Systems
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Multicarrier (MC) and Code Division Multiple Access (CDMA) have gained considerable interest due to their considerable performance.

**OFDM (Orthogonal Frequency Division Multiplexing)**

- Robust against multi-path propagation effects
- Reduced system complexity due to equalization in the frequency domain
- Increase of spectral efficiency
- Efficient modulation algorithm available (IFFT, FFT)
- No continuous spectrum required
- The capability of narrow-band interference rejection
Introduction

Code Division Multiple Access (CDMA)

- Allows multiple users to share same bandwidth at the same time
- An ability to reduce user’s signal power during transmission using a power control algorithm
- Extended battery life because of effective power control
- No guard bands or guard times are typically required relative to TDMA and FDMA

MC-CDMA

As a combination of OFDM and CDMA techniques, it combines the advantages of both OFDM and CDMA
Objective of this work

The quality of multiple access interference (MAI), which can be improved by using channel estimation and data estimation of all active users, effects considerably the performance of PIC detector.

So obviously it is necessary to make excellent joint data and channel estimation for initialization of PIC detector.

Therefore, data and channel estimation performance is obtained in the initial stage has a significant relationship with the performance of PIC.
We propose a joint MAP channel estimation and data detection technique based on the Expectation Maximization (EM) method with parallel interference cancelation (PIC) for downlink multi-carrier (MC) code division multiple access (CDMA) systems in the presence of frequency selective channels.

The EM algorithm derived estimates the complex channel parameters of each subcarriers iteratively and generates the soft information representing the data a posterior probabilities.

The MAP-EM approach considers the channel variations as random processes and applies the Karhunen-Loeve (KL) orthogonal series expansion.
Transmission of MC-CDMA signals from the base station to mobile stations forms the downlink transmission.

In the downlink applications, all the signals arriving from the base station to specific user propagate through the same channel. Therefore, channel estimation methods that is developed for OFDM systems can be applicable for downlink application of MC-CDMA systems.
**Signal Model**

$b^k$ the QPSK modulated symbols that would be send for $k$th user $k=1,...,K$

$K$: is the number of mobile users which are simultaneously active

The base station spread the data by means of specific orthogonal spreading sequences

$\mathbf{c}^k = (c_1^k, c_2^k, ..., c_{N_c}^k)^T$

$\mathbf{c}^k b^k$

The spreaded sequences of all users are summed together to form the input sequences of the OFDM block

After summation process, OFDM modulator block takes inverse discrete Fourier transform (IDFT) and inserts cyclic prefix (CP) of length equal to at least the channel memory (L).

**Transmitter**
Signal Model (Cont.)

At the receiver, CP is removed and DFT is then applied to the received discrete time signal to obtain the received vector

\[ R = \mathcal{H}C b + W \]

\[ C = [c^1, \ldots, c^K] \rightarrow N_c \times K \text{ spreading code matrix} \]

\[ b = [b^1, \ldots, b^K]^T \rightarrow K \times 1 \text{ vector of the transmitted symbols by the } K \text{ users} \]

\[ \mathcal{H} \rightarrow N_c \times N_c \text{ diagonal channel matrix whose elements representing the fading of the sub-carriers} \]

\[ W \rightarrow N_c \times 1 \text{ zero-mean, i.i.d. Gaussian vectors that model additive noise in the } N_c \text{ tones} \]

Note that due to the orthogonality property of spreading sequences \[ C^T C = I \]
KL expansion of the channel

\[ H = [H_1, H_2, \ldots, H_{N_c}]^T \] the correlated channel coefficients corresponding frequency response of the channel between the transmit and the receive antenna

The autocorrelation matrix

\[ C_H = E[HH^\dagger] \quad \text{decomposed as} \quad C_H = \Psi \Lambda \Psi^\dagger \]

\[ H = \Psi G \]

\[ \Psi = [\psi_1, \psi_2, \ldots, \psi_{N_c}] \quad \text{\(\psi_i\)'s are the orthonormal basis vectors} \]

\[ G = [G_1, \ldots, G_{N_c}]^T \quad \text{\(G_i\) is the vector representing the weights of the expansion} \]

\[ \Lambda = E\{GG^\dagger\} \]
KL expansion

Why ???

The fact that only the eigenvectors diagonalize $C_H$ leads to the desirable property that the KL coefficients are uncorrelated.

Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity.

Thus, the channel estimation problem in this study is equivalent to estimating the i.i.d. Gaussian vector $G$ namely, the KL expansion coefficients.
MAP channel estimation

\[ R = \mathcal{H}Cb + W \]

\[ \hat{R} = \text{diag}(Cb) \Psi G + W \]

In the MAP estimation approach, we choose \( \hat{G} \) to maximize the posterior PDF

\[ \hat{G} = \arg \max_G p(G|R) \]

Obtaining MAP estimate of \( \hat{G} \) is a complicated optimization problem and does not yield to a closed form solution.

Solution of such problems usually requires iterative methods. \( \text{EM algorithm} \)

EM algorithm inductively reestimate \( \hat{G} \) so that a monotonic increase in the a posteriori conditional pdf is guaranteed.
MAP channel estimation

The monotic increase is realized via the maximization of the auxiliary function

\[ Q(G|G^{(q)}) = \sum_b p(R, b, G^{(q)}) \log p(R, b, G) \]

Given the received signal \( R \), the EM algorithm starts with initial value of \( G^{(0)} \) of the unknown channel parameters \( G \).

The \((i+1)\) th estimate of \( G \) is obtained by the maximization step

\[ G^{(i+1)} = \arg \max_G Q(G|G^{(i)}) \]

\[ \log p(R, b, G) = \log p(b | G) + \log p(R | b, G) + \log p(G). \]

\[ p(R|b, G) \sim \exp \left[ -(R - \text{diag}(Cb) \Psi G)^\dagger \Sigma^{-1} (R - \text{diag}(Cb) \Psi G) \right] \]

\[ p(G) \sim \exp(-G^\dagger \Lambda^{-1} G) \]

Taking derivatives with respect to \( G \) and equating the resulting equations to zero

\[ \sum_b p(R, b, G^{(q)}) (\Psi^\dagger \text{diag}(b^\dagger C^T) \Sigma^{-1} (R - \text{diag}(Cb) \Psi G) - \Lambda^{-1} G) = 0 \]
Receiver Structure

\[ \hat{G}^{(q+1)} = (T^{(q)^\dagger} T^{(q)} + \Sigma \Lambda^{-1})^{-1} T^{(q)^\dagger} R \]

\[ T^{(q)} = \text{diag}(C \Gamma^{(q)}) \Psi, \]

\[ \Gamma^{(q)} = [\Gamma^{(q)}(1), \Gamma^{(q)}(2), \ldots, \Gamma^{(q)}(K)] \text{ represents the a posteriori probabilities of the data} \]

\[ \Gamma^{(q)} \text{ can be computed for QPSK signaling as follows} \]

\[ \Gamma^{(q)} = \frac{1}{\sqrt{2}} \tanh \left[ \frac{\sqrt{2}}{\sigma^2} Re(\hat{Z}^{(q)}) \right] + \frac{j}{\sqrt{2}} \tanh \left[ \frac{\sqrt{2}}{\sigma^2} Im(\hat{Z}^{(q)}) \right] \]

\[ \hat{Z}^{(q)} = C^T (\hat{\mathcal{H}}^{(q)^\dagger} \mathcal{H}^{(q)} + \sigma^2)^{-1} \mathcal{H}^{(q)^\dagger} R \]
Complexity

**Truncation property**

The truncated basis vector $G$ can be formed by selecting $r$ orthonormal basis vectors among all basis vectors that satisfy $C_H \Psi = \Psi \Lambda$

$$\Lambda_r = diag \{ \lambda_1, \lambda_2, \cdots, \lambda_r \}$$ by ignoring the trailing $N_c - r$ variances $\{ \lambda_i \}_{i=r+1}^{N_c}$.

- **the traditional LMMSE estimation**
  $$\hat{H} = C_H \left[ C_H + \Sigma (\text{diag}(C_D))^\dagger \text{diag}(C_D))^{-1} \right]^{-1} [\text{diag}(C_D)]^{-1} R$$
  "$O(N_c^3)$" computational complexity

- **the KL based approach**
  $$\hat{H}^{(q+1)} = \Psi (T^{(q)}_r T^{(q)}_r + \sum \Lambda^{-1})^{-1} T^{(q)}_r R$$

- **the low-rank approach**
  $$\hat{H}^{(q+1)} = \Psi_r (T^{(q)}_r T^{(q)}_r + \sum \Lambda^{-1}_r)^{-1} T^{(q)}_r R$$

$\Psi_r$ and $T_r$ is an $N_c \times r$ matrix which can be formed by omitting the last $N_c - r$ columns of $\Psi$ and $T$ respectively.
Simulations

The orthogonal Gold Sequence code selected as spread code and processing gain equals to the number of subcarriers.

The number of active users $K$, is equal to the length of the spreading code $N_c = 128$.

Pilot Insertion Rate was chosen as (PIR) = 1:8.

Proposed approach compared with:

- LS channel estimation - MMSE detection
- LS channel estimation - MMSE-PIC detection
- LMMSE channel estimation - MMSE detection
- LMMSE channel estimation - MMSE-PIC detection
Simulations

The proposed EM-MAP algorithm

- Outperforms the LS as well as LMMSE techniques
- Approaches the allpilot estimation case for higher Eb/No values.
Simulations

BER performance of the proposed receiver structure is much better than the combined MMSE-PIC receiver in the case of LS, LMMSE.

Approaches the performance of the all-pilot and perfect channel estimation cases.
Simulations

Optimal truncation property of the KL expansion

It is possible to obtain an excellent approximation with a relatively small number of KL coefficients.
**Conclusion**

In this work we have presented an efficient EM-MAP receiver structure for downlink MC-CDMA systems.

This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing MPSK modulation scheme with additive Gaussian noise.

Simulation studies have indicated that the MSE and BER performance of the proposed algorithm well over the conventional algorithms and approaches the perfect estimation case by iterative improvement.

It was demonstrated that complexity of the channel estimator could be reduced noticeably by using the optimal truncation property of the KL expansion.
Thanks for your attention

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