EXPERIMENT #2
DISCRETE SYSTEMS
DUE: Feb. 15, 1995

Objective
The purpose of this experiment is to analyze and verify various properties of discrete-time systems.

Prelab Preparation
Before going to the lab you are expected to do the following:

1. Review the properties of discrete-time systems.
2. Do the prelab exercises.
3. Predict the results of the experimental steps described below.

SECTION 1: CONVOLUTION SUM

Procedure

1. Generate a signal with a 100 samples, the first 50 samples being equal to one and the rest equal to zero. Consider this signal the impulse response of a linear time-invariant (LTI) system.

2. Generate an input signal that is also 100 samples long, the first 50 samples being equal to one and the rest zeros. Compute the output of the system using the convolution sum. How does your result agree with theoretical results? How are the lengths of the impulse response, the input and the output related?

3. Generate a unit impulse signal of length 50. Compute the response to this signal of the system, with the impulse response in 1, using convolution. How does your result compare with the impulse response? Delay the input unit impulse signal by 10 samples and compute the output using the convolution sum, how does this output compare with the one obtained before?

4. Generate a zero-average Gaussian noise signal $x(n)$ of length 60. To make sure that it has zero average, calculate its average and subtract it from the signal. Convolve this signal with a square signal $y(n)$ that has a length of 60 and repeats every 4 samples, and call the resulting signal $z(n)$. Would you get the same result if you had convolved $y(n)$ with $x(n)$, i.e., is the convolution sum commutative?

5. Calculate the sum of the Gaussian noise $x(n)$, the square signal $y(n)$ and the convolution of these signals $z(n)$. How are these sums related?

6. Calculate the average of $x(n)$, how does it relate to the average of $x(n)$?
Discussion

• Derive a general relation between the length of the result of a convolution and the lengths of the sequences being convolved.

• Use the Fourier transform of the convolution sum to explain the relation of the sums of the different signals in 5. Could this relation be used as a check on the correctness of a convolution sum?

• A noise signal can be expressed as \( x(n) = \alpha + v(n) \), where \( \alpha \) is the average, and \( v(n) \) is a zero-average noise signal. If you convolve \( x(n) \) with \( y(n) \) to get \( z(n) \), what would be the average of \( z(n) \) according to the results in 6? Explain. What property of the convolution are you using?

SECTION 2: LINEARITY

Procedure

1. Generate signals (in SIGLAB notation) \( s = 0.1 \ast \text{rand}(150) \) and \( v = 0.1 \ast \text{tri}(4,150) \) and consider them small disturbances. Create then a unit pulse signal \( x(n) \) with 150 samples, where the first 10 and last 10 are zeros and the rest are ones. Then generate two noisy pulses:

\[
\begin{align*}
    x_1(n) &= x(n) + s(n) \\
    x_2(n) &= x(n) + v(n)
\end{align*}
\]

2. Pass the noisy signals \( x_1(n) \) and \( x_2(n) \) through the system defined by the equation

\[
y(n) = 0.33[x(n) + x(n - 1) + x(n - 2)]
\]

where \( x(n) \) is the input and \( y(n) \) is the output. What does this system do?

3. Pass the sum \( x_1(n) + x_2(n) \) through the system above.

4. Does the result of the step above equal the sum of the outputs generated in step 3? If so, does this prove that the system is linear? Why?

5. Determine theoretically the impulse response \( h(n) \) of the system in 2, and calculate the convolution of \( x_1(n) \) and \( h(n) \). How does this compare with the output corresponding to \( x_1(n) \) obtained before in part 2? Is the system in part 2, a finite impulse response (FIR) or infinite impulse response (IIR) system? What type of filter (i.e. lowpass, highpass, bandpass, etc.) is it?

Discussion

• What is the relation between the coefficients of a non-recursive filter (like the one given in step 2 above) and its impulse response?
Can the output of a non-recursive filter always be computed using convolution? Explain.

Suppose you wished to calculate the output of the system in 2 using FFTs, how would you do it? What would be the minimum length of the FFTs? Why?

SECTION 3: TIME-ININVARIANCE

Procedure

1. Consider a system defined by the equations
   \[ y(n) = x(n) + x(n - 1); \quad x(n) \geq 0 \]
   \[ y(n) = x(n) - x(n - 1); \quad x(n) < 0 \]

2. Create two input signals. One, \( x_1(n) \), has a length of 100 samples, and the first 90 samples correspond to a square signal that repeats every 4 samples and the rest are zeros. The other signal \( x_2(n) \) is also 100 samples long, but the first 90 samples correspond to a triangular signal that repeats every 4 samples.

3. Compute the outputs corresponding to each of the two input signals in step 2.

4. Compute the output corresponding to the sum of the signals generated in step 2. Comparing this output with the sum of the outputs obtained in step 3, is the process linear? Explain.

5. Delay the input signal \( x_1(n) \) by 10 samples and pass the delayed signal through the system defined in 1. How does the resulting output compare with the one obtained in 3 for \( x_1(n) \) as input? Is the system defined in 1 time-invariant? Explain.

Discussion

- If you wanted to write the equation in step 1 as
  \[ y(n) = x(n) + a(n)x(n - 1) \]
  what would \( a(n) \) be equal to?

SECTION 4: RECURSIVE SYSTEMS

Procedure

1. Given the recursive system
   \[ y(n) = x(n) + 1.5x(n - 1) - 0.9y(n - 1) \]
compute recursively its impulse response \( h(n) \) by setting the input equal to a unit impulse and the initial conditions equal to zero. What is the length of the impulse response?
2. Compute the step response $s(n)$ of the above system.

3. Do the computed impulse and step responses match the theoretical impulse and step responses? Explain any discrepancy.

4. How would you obtain the impulse response $h(n)$ from the step response $s(n)$? Explain. What is the system property that allows you to do that?

5. Consider the sinusoidal steady state response of a recursive system. Create an input signal $x(n) = 1 + \cos(\pi n/4)$ of length 200. Let the system be represented by the input/output equation

$$y(n) = x(n) + 1.5x(n - 1) - \alpha y(n - 1)$$

and $\alpha$ changes from 0.6 to 1.4 in 0.2 intervals. Compute the output for each of these cases and the input indicated above. Comment on the transient and the steady state behavior for the cases when the alpha parameter is less or equal to 1. What happens when alpha is larger than one? What is the reason for this behavior?

**Discussion**

- Can a recursive system have a finite impulse response? If so, give an example.

- How would you calculate the magnitude of the steady state sinusoidal components? Consider the case in 5, determine the amplitude of each of the sinusoidal components (a constant is a cosine of zero frequency) when the output reaches steady state.
Procedure/Discussion

1. The output of an echo system is

\[ y(n) = x(n) - 0.25x(n-2) \]

where \( x(n) \) is an acoustic signal (e.g. speech or music) that is delayed, attenuated and added to create \( y(n) \) as shown. Compute the impulse response of the echo system. Is this an FIR or an IIR system?

2. To recover the original acoustic signal, \( x(n) \), one can filter \( y(n) \) with a so called inverse filter. The inverse filter has a transfer function that is equal to the reciprocal of the transfer function of the echo system. Compute the impulse response of the inverse filter for the echo system given in 1 above. What is the length of this impulse response?

3. Suppose the acoustic signal is a pure tone, \( x(n) = \cos(\pi n/2) \). Compute \( y(n) \) and the inverse filter output using convolution. Restrict the length of the impulse response of the inverse filter to 5, 10, 20 and 100 (i.e. only consider the first 5, 10, 20, and 100 samples of the impulse response, make the rest zeros). How does the length restriction affect the results?

4. Suppose the output of the echo system is corrupted by a random noise \( v(n) \), i.e., the noisy output is \( z(n) = y(n) + v(n) \). The random noise is generated by creating first a random sequence and then multiplying it by a small number, let us say 0.01. How would you recover the original pure tone using the filter in section 2, step 2, and the inverse filter? Explain and justify your procedure.