EXPERIMENT #3
FREQUENCY ANALYSIS
DUE: July 10, 1995

Objective
The purpose of this experiment is to analyze various properties of the Fourier Transform, and its implementation using the Fast Fourier Transform.

Prelab Preparation
Before going to the lab you are expected to do the following:

1. Familiarize yourself with the Monarch commands indicated in the TA hints.
2. Do the prelab exercises.
3. Predict the results of the experimental steps described below.

SECTION 1: FREQUENCY RESOLUTION AND CONCENTRATION

Procedure

1. Generate a square signal that repeats every 16 samples, has a duty cycle of 1/2 and has a length of 256. Consider this signal part of a periodic square wave of period 16.

2. Using the FFT function find the Fourier Transform of the signal in step 1, and plot its magnitude and phase. What frequencies do each of the peaks in the magnitude plot correspond to? Why do the magnitude and phase values occur in a finite set of frequencies?

3. Generate a new signal by taking the first 16 values of the signal in 1, and consider it one period of a periodic square wave of period 16.

4. Compute the FFT of the signal in 3 and plot the corresponding magnitude and phase. How do the magnitude and phase obtained in 2 compare with the magnitude and phase obtained here? Which one gives a better frequency resolution?

5. Generate a square signal that repeats every 64 samples, has a duty cycle of 1/2 and length of 256. Consider this signal part of a periodic square wave of period 64. Compute its FFT and plot its magnitude and phase. Compare these results with the magnitude and phase obtained in 2. How do the frequency concentrations change?

6. Generate two aperiodic signals: \( x(n) \) consists of 5 ones and 251 zeros, and \( y(n) \) consists of 5 ones and 3 zeros. Compute the FFT of each of these signals and compare their magnitude and phase plots. What is different between \( x(n) \) and \( y(n) \)? Do they have the same Discrete Time Fourier Transform (DTFT)? What is the effect of the zeros in calculating the FFT?
Discussion

- Discuss the advantages of using several periods to calculate the FFT of a periodic signal and what changes need to be made to adjust the magnitude.

- Comment on how the duration in time of a signal and its concentration in the frequency domain are related.

- To improve the frequency resolution when calculating the FFT of periodic and aperiodic signals, we use different techniques. In the periodic case, we consider several periods and in the aperiodic case we pad the signal with zeros. Explain why we cannot use the padding with zeros in the periodic case.

SECTION 2: SHIFTING IN TIME AND PHASE

Procedure

1. Generate a pulse signal \( x(n) \) that has 5 ones and 251 zeros. Shift \( x(n) \) by 2 samples and call the resulting signal \( y(n) \). Compute the FFT of the two signals and plot their magnitudes and phases. Compare their magnitude and phase plots. Can you determine from the phase plots the effect of shifting the signal? Theoretically what is the phase difference?

2. Generate the 128-point signal \( x(n) = \cos(\pi n/16 + \theta) \) for the following values of \( \theta : 0, \pi/2, \pi \). You have then 3 cosine signals each with different phase.

3. Use the FFT to compute the magnitude and phase spectra of each of the 3 signals in 2. Plot the magnitude and phase for each signal. How do the phase plots compare with the theoretical results? Why do you think this is happening? Clue: display the FFT values and look at the values not at the harmonic frequency.

4. Develop a simple algorithm to "clean-up" the phase of the FFT of the cosine: when the magnitude is not significant the phase should be set to zero. Plot the clean phases of the 3 cosines signals generated in 2.

Discussion

- What seems to be the difficulty in looking at phase plots? Is the difficulty due to the way the arctan is calculated? If there are no poles or zeros of the z transform of the signal on the unit circle, should the corresponding phase be continuous?

SECTION 3: MODULATION

Procedure

1. Generate a 256-point signal \( x(n) = u(n) - u(n - 65) \) and find its magnitude and phase spectra using the FFT.
2. Multiply \( x(n) \) by the sinusoid \( s(n) = \cos(0.5\pi n) \) of length 256. Compute the magnitude and phase spectrum of the resulting signal using the FFT. How do they relate to the magnitude and phase spectra found in step 1?

3. Multiply \( x(n) \) by \( s(n) = \sin(0.5\pi n) \) and compute the magnitude and phase spectra of the resulting signal using the FFT. How do they relate to the magnitude and phase spectra found in steps 1 and 2?

4. Multiply \( x(n) \) by \( s(n) = e^{j0.5\pi n} \) and find the magnitude spectrum of the resulting signal. How do the magnitude spectra in steps 2, 3, and 4 differ and why?

**Discussion**

- Discuss the effect of multiplying a signal by a sinusoid on the frequency content of the signal. What is the difference between using a complex sinusoid and a real sinusoid in terms of the resulting frequency content?

- What is the difference between modulation using a cosine and using a sine?

**SECTION 4: FILTERING**

**Procedure**

1. Generate a pulse signal \( x(n) \) having 50 ones and 50 zeros. To "distort" this clean pulse add to \( x(n) \) 100 samples of random noise multiplied by 0.1 (i.e. in SIGLAB notation: add 0.1 * rand(100)), and to the resulting signal add a cosine of amplitude 0.3, period 16, and phase 0. Call the resulting signal \( z(n) \). In the following steps we will attempt to get rid of the distortions and recover the original pulse \( x(n) \) using different filters.

2. Consider the impulse response \( h(n) \) of an FIR filter with values

\[
1, 2, 3, 4, 5, 4, 3, 2, 1
\]

Calculate its magnitude and phase responses using an FFT of length 256. What type of frequency discriminating filter is this? Is the phase linear in the passband?

3. Consider the impulse response \( g(n) \) of another FIR filter with values

\[
1, 0, 0, 0, 0, 0, 0, 0, 0, 1
\]

Calculate its magnitude and phase responses using an FFT of length 256. This FIR filter is a notch (band-stop) filter. What are the notch frequencies? Is the phase linear in the passband?

4. Determine the input/output of the FIR filter given in 2, and pass \( x(n) \) through the filter to get as output \( y(n) \). What was the effect of the filter on \( x(n) \)?
5. Compute the 256-point FFT of \( h(n) \), and of \( z(n) \). Multiply the FFTs and use the inverse FFT to obtain the output of the filter. Is this output real? If the imaginary part of the filter output is negligible consider only the real part of the output and call it \( y_1(n) \). How do \( y(n) \) and \( y_1(n) \) compare?

6. Plot the magnitudes of the FFTs of \( x(n) \), \( h(n) \) and \( y_1(n) \), and determine in the frequency domain what the effect of filtering with this filter is.

7. Compute the 256-point FFT of \( g(n) \). Multiply this FFT with the FFT of \( z(n) \) and use the inverse FFT to obtain the output of the filter. The imaginary part of the filter output is negligible thus consider only the real part of the output and call it \( y_2(n) \).

8. Plot the magnitudes of the FFTs of \( z(n) \), \( g(n) \) and \( y_2(n) \), and determine in the frequency domain what the effect of filtering with this filter is.

9. Use the given FIR filters to develop a procedure to minimize the distortion on \( x(n) \) due to both the additive noise and sinusoid. Show the results of your procedure.

Discussion

- Discuss the connection between the convolution sum and its length with the filtering procedure in the frequency domain implemented using the FFT.

- If you were given an IIR filter would you do the filtering using FFTs? Why or why not.
POSTLAB #3
SCRAMBLING SOCIAL SECURITY NUMBERS
DUE: July 10, 1995

Procedure/Discussion In practice, there are situations when for security reasons you might want to scramble a signal so that only a limited number of persons be able to have access to it. The following is a simple example of a possible way to do the scrambling. Suppose you want to send your social security number (ss#), or your pin for a bank account, by e-mail and you want to protect it from people other than those you want to communicate it to, and who know your process or key to be able to decipher the information. Thus, consider a discrete-time signal $x(n), 0 \leq n \leq 8$ whose values are the integers of your ss#. To scramble it, you follow the following procedure:

1. Interpolation: To confuse the hackers you want to add zeros in between the digits of your ss# and then smooth it (or interpolate it), thus you create first a signal

$$s(n) = \sum_{m=0}^{q=2} h(m)y(n-m)$$

where $y(n), 0 \leq n \leq 127$ is created by including three zeros in between the social security integers, i.e.,

$$y(n) = \begin{cases} 
\frac{x(n-4)}{4} & n = 4, 8, \cdots, 36 \\
0 & \text{otherwise}
\end{cases}$$

and the $\{h(n)\}$ correspond to the impulse response of a moving-average FIR filter

$$H(z) = \sum_{n=0}^{2} \alpha^n z^{-n}$$

with $\alpha = 0.8$. Plot the original signal $x(n)$ (your social security number), $y(n)$ (the social security number with zeros included between the ss digits) and $s(n)$ the interpolated signal. Can you recognize your ss#?

2. "Noise" addition: To make sure that the hackers cannot decipher your ss# you proceed now to add a modulated cosine signal of high frequency so that the signal looks "scrambled", i.e., generate a signal $ss(n)$ as

$$ss(n) = \begin{cases} 
\frac{s(n) + 5 \cos(\pi n) \sin(\pi n/16)}{4} & 0 \leq n \leq 47 \\
0 & \text{otherwise}
\end{cases}$$

Plot the superscrambled signal $ss(n), 0 \leq n \leq 127$, can you recognize your ss#?

In practice, the above scrambling is nothing more than a simple example of the denoising and inverse filtering problems that occur quite frequently in signal processing problems (find out where!).
You want now to unscramble the \(ss(n)\) signal using the techniques learned in this lab.

1. Denoising: to find out what type of filter to use, calculate the FFT of \(ss(n)\), as well as those of the \(y(n)\) and the "noise" \(\eta(n) = 5 \cos(\pi n) \sin(\pi n/16)\). To get rid of \(\eta(n)\) use the following filter (what type of filter is this?)

\[
y(n) = x(n) + 2x(n - 1) + x(n - 2) - 1.2y(n - 1) - 0.45y(n - 2)
\]

Plot the magnitude spectra of the three signals \(ss(n), y(n)\) and \(\eta(n)\); what type of signals are these? Plot then the result of filtering \(ss(n)\) with the filter you chose, \(\hat{s}(n)\) and calculate its magnitude spectrum and plot it. Did you use the right filter, i.e., did you get rid of the noise? How would you know if you did or not?

2. Inverse filtering: after getting rid of the noise we then need to get rid of the effect of the smoothing caused by the MA filter. What type of filter, FIR or IIR, would you use for that? What would be its transfer function? After you decide the type of filter to use, calculate its output \(\hat{y}\) (that is, filter \(\hat{s}(n)\) with your chosen filter). How does signal look like? Explain the significance of \(\alpha\) by choosing a new value of \(\alpha = 1\) (the filter would then be like the MA seen in class) and trying to find the inverse filter. What happens? Explain.

3. Decimation: If things have gone right, you must have gotten rid of the noise and the effect of the interpolating, so now you need to get rid of the zeros that you put between the \(ss\) digits or to decimate. So get every fourth value of \(\hat{y}(n)\) and compare it with your \(ss\#\). How close is it? What would you do differently to get a final result that is closer to your \(ss\#\)?

\underline{Warning: Do not use this technique in real life, because most hackers know signal processing and could know this technique (besides, now that the technique is in print we have gotten the cat out of the bag!).}
Pre-Lab Exercises #4 — Due: Mar. 1, 1995

**Prob. 1** Consider the bandpass filter represented by the difference equation

\[ y(n) = -0.25 y(n-4) + z(n) - z(n-2) \]

where \( z(n) \) is the input and \( y(n) \) the output of the filter.

1. Find the transfer function \( H(z) \) of the filter.

2. Locate in the \( z \)-plane the poles and zeros of the filter. Use this information to explain the reason this is a bandpass filter.

3. What are the values of the magnitude response at \( \omega = 0, \pi \)?

**Prob. 2** You wish to use the FFT to calculate the frequency response of an IIR filter with transfer function

\[ H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \]

and \( N \geq M \). Explain how to do it.

**Prob. 3** Given the following transfer functions

\[ H_1(z) = \frac{z}{z - 0.9} \]
\[ H_2(z) = \frac{z}{z + 0.9} \]
\[ H_3(z) = \frac{z^2}{z^2 - 0.9z + 0.81} \]
\[ H_4(z) = \frac{z^2 + 1}{z^2 + 0.81} \]

Locate the poles and zeros of each transfer function and from that information determine the type of filter and carefully sketch the magnitude responses.

**Prob. 4** Give the transfer function of a double bandpass IIR filter that blocks the DC and high frequency (\( \omega = \pi \)) components of the input, has very sharp peaks at \( \omega_1 = \pm \pi/3 \) and \( \omega_2 = \pm 2\pi/3 \), and is stable.

**Prob. 5** The frequency response of a filter is

\[ H(e^{j\omega}) = 2e^{-2j\omega} \quad \pi/3 \leq |\omega| \leq 2\pi/3 \]
\[ 0 \quad \text{otherwise in } (-\pi, \pi) \]

1. What type of filter is this? Is this a real or an ideal filter?

2. Determine the steady state response of the above filter when the input is

\[ z(n) = 2 + 4.5 \cos(\pi n/4) + 9 \cos(3\pi n/8) \]