Pre-Lab Exercises #4 — Due: Mar. 1, 1995

Prob. 1 Consider the bandpass filter represented by the difference equation

\[ y(n) = -0.25 y(n-4) + x(n) - x(n-2) \]

where \( x(n) \) is the input and \( y(n) \) the output of the filter.

1. Find the transfer function \( H(z) \) of the filter.
2. Locate in the z-plane the poles and zeros of the filter. Use this information to explain the reason this is a bandpass filter.
3. What are the values of the magnitude response at \( \omega = 0, \pi \)?

Prob. 2 You wish to use the FFT to calculate the frequency response of an IIR filter with transfer function

\[ H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \]

and \( N \geq M \). Explain how to do it.

Prob. 3 Given the following transfer functions

\[ H_1(z) = \frac{z}{z - 0.9} \]
\[ H_2(z) = \frac{z}{z + 0.9} \]
\[ H_3(z) = \frac{z^2}{z^2 - 0.9z + 0.81} \]
\[ H_4(z) = \frac{z^2 + 1}{z^2 + 0.81} \]

Locate the poles and zeros of each transfer function and from that information determine the type of filter and carefully sketch the magnitude responses.

Prob. 4 Give the transfer function of a double bandpass IIR filter that blocks the DC and high frequency (\( \omega = \pi \)) components of the input, has very sharp peaks at \( \omega_1 = \pm \pi/3 \) and \( \omega_2 = \pm 2\pi/3 \), and is stable.

Prob. 5 The frequency response of a filter is

\[ H(e^{j\omega}) = 2e^{-j2\omega} \quad \pi/3 \leq |\omega| \leq 2\pi/3 \]
\[ 0 \quad \text{otherwise in } (-\pi, \pi) \]

1. What type of filter is this? Is this a real or an ideal filter?
2. Determine the steady state response of the above filter when the input is

\[ x(n) = 2 + 4.5 \cos(\pi n/4) + 9 \cos(3\pi n/8) \]