16.5 Integrals in Cylindrical and Spherical Coordinates
Relation between Cartesian and Cylindrical Coordinates:

Each point in 3-space is represented using \(0 \leq r \leq \infty, 0 \leq \theta \leq 2\pi, -\infty \leq z \leq \infty\).

\[
x = r \cos \theta, \quad y = r \sin \theta, \quad z = z.
\]
Example 1. Describe the solid below using cylindrical coordinates.
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Answer:

\[ 0 \leq r \leq 6, \quad 0 \leq z \leq 4, \quad 0 \leq \theta \leq \frac{\pi}{6} \]
Integration in Cylindrical Coordinates:

When integrating in cylindrical coordinates put $dV = r dr d\theta dz$. 
Example 2. Find the mass of the solid below if its density is 1.2 grams / cm$^3$. 
Example 3. A water tank in the shape of a hemisphere has radius \(a\); its base is its plane face. Find the volume \(V\), of water in the tank as a function of \(h\), the depth of the water.

Why do we write \(r^2 + z^2 = a^2\)?
Volume of water = \( \int_{0}^{\theta} \int_{0}^{r} \int_{0}^{h} r \, dr \, dz \, d\theta \)
Spherical Coordinates

Each point in 3-space is represented using $0 \leq \rho \leq \infty$, $0 \leq \phi \leq \pi$, $0 \leq \theta \leq 2\pi$

$$x = \rho \sin \phi \cos \theta$$
$$y = \rho \sin \phi \sin \theta$$
$$z = \rho \cos \phi$$

Observe that $\rho^2 = x^2 + y^2 + z^2$
When computing integrals in spherical coordinates, put $dV = \rho^2\sin\phi d\rho d\phi d\theta$.
Example 4. Use spherical coordinates to derive the formula for the volume of a solid sphere of radius $a$. 
Exercise 12-18. Choose coordinate system and set up a triple integral, including limits of integration, for a density function $f$ over the region.
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A piece of sphere; angle at the center is $\frac{\pi}{3}$. 